Toward Feedback Control of Densities in Nonlinear Systems

From its very inception, technological applications have guided two central questions in control theory: “how to control,” and perhaps, more fundamentally, “what to control.” Early applications ushered much of the traditional control theory to focus on manipulating signals or trajectories, where uncertainties were handled in the spirit of “control with uncertainties,” as opposed to “control of uncertainties.” In this talk, we will point on how many modern applications motivate directly controlling uncertainties modeled via joint probability density functions supported over the state space. The same mathematical formulation applies in purely deterministic setting, where the density to be manipulated represents a physical population such as robotic swarm, neuronal ensemble, or concentration of chemicals. Designing feedback control in this context is an emergent research area witnessing fast development within the systems-control-robotics communities. This talk will present research results on how to steer densities on the state space in a desired manner subject to controlled nonlinear trajectory-level dynamics. We will show that in certain nonlinear settings of practical interest, the problem can be tackled by bringing together tools from stochastic control, optimal mass transport and Schrödinger bridges. Numerical examples will be given to illustrate the theoretical results.